THREE–BODY RESONANCES IN FRAMEWORK OF THE FADDEEV CONFIGURATION SPACE APPROACH[1]

E. A. Kolganova, A. K. Motovilov

Joint Institute for Nuclear Research, Dubna, Russia
e-mail: kea@thsun1.jinr.ru, motovilv@thsun1.jinr.ru

Abstract. Algorithm, based on explicit representations for analytic continuation of the T-matrix Faddeev components on unphysical sheets, is worked out for calculations of resonances in the three-body quantum problem. According to the representations, poles of the T–matrix, scattering matrix and resolvent on unphysical sheets, interpreted as resonances, coincide with those complex energy values where appropriate truncations of the scattering matrix have zero as eigenvalue. Scattering amplitudes on the physical sheet, necessary to construct scattering matrix, are calculated on the basis of the Faddeev differential equations. Effectiveness of the algorithm developed is demonstrated for example of searching for resonances in the system *nnp* and in a model three-boson system.

We make a numerical test of the approach proposed in Ref. [1] to treat the three–body resonances in the case of pairwise interactions falling off in coordinate space not slower than exponentially. This approach is based on a construction of explicit representations for the Faddeev components $M_{\alpha\beta}(z)$, $\alpha, \beta = 1, 2, 3$, of the three–body T-matrix $T(z)$ as well as for the scattering matrices $S(z)$ and resolvent $R(z)$ continued on unphysical sheets of the energy z plane. For the sheets, the notation Π_l is used with l, an enumerating multi–index (see [1]). The representations constructed demonstrate a structure of kernels of the above operators after continuation and give new capacities for analytical and numerical studies of three–body resonances. In particular the representations for analytic continuation $M(z)|_{\Pi_l}$ of the matrix $M(z) = \{M_{\alpha\beta}(z)\}$ on the sheet Π_l with details omitted read

$$M|_{\Pi_l} = M|_{\Pi_0} - Q_l^{\dagger}\mathbf{J}^{\dagger}AS_l^{-1}\mathbf{J}Q_l$$

The operator $Q_l(z)$ and the "transposed" one, $Q_l^{\dagger}(z)$ are obviously constructed of the matrix $M(z)$ taken on the physical sheet Π_0. $A(z)$ is a number matrix, an entire function of $z \in \mathbb{C}$. By $S_l(z)$ we understand a truncation (depending essentially on l) of the total three–body scattering matrix $S(z)$. Operators $\mathbf{J}(z)$ and $\mathbf{J}^{\dagger}(z)$ realize a restriction of kernels of the operators $Q_l(z)$ and $Q_l^{\dagger}(z)$ on the energy shells, respectively, in first and last momentum variables. So that the products $Q_l^{\dagger}\mathbf{J}^{\dagger}$ and $\mathbf{J}Q_l$ have half–on–shell kernels. Representations [1] for analytical continuation of the three–body scattering matrices and resolvent follow immediately from the representations above for $M(z)|_{\Pi_l}$.

As follows from the representations constructed, the nontrivial (i.e. differing from the poles at the discrete spectrum eigenvalues of the three–body Hamiltonian) singularities of the T–matrix, scattering matrices and resolvent situated on an unphysical sheet Π_l, are singularities of the inverse truncated scattering matrix $S_l^{-1}(z)$. Therefore, the resonances on the sheet Π_l considered as poles of the T–matrix, scattering matrix and resolvent continued on Π_l are those values of the energy z for which the matrix $S_l(z)$ has zero

[1] LANL E-print `nucl-th/9702037`. To appear in Proc. of Intern. Conf. on Computational Modelling and Computing in Physics (Dubna, September 16–21, 1996).

Figure 1: Surface of the function $|S_{01}(z)|$ in the model system of three bosons with the nucleon masses. The potential $V^G(r)$ is used with the barrier $V_b = 1.5$ MeV. Position of the resonance $z_{\rm res}(3{\rm B})$ corresponds to the minimal (zero) value of $|S_{01}(z)|$.

as eigenvalue. Thereby, to search for the resonances situated on a certain unphysical sheet Π_l, one can apply any method allowing to compute analytical continuation on the physical sheet of the elastic scattering, rearrangement or breakup amplitudes necessary for construction of respective $S_l(z)$. In particular such is the algorithm developed for $(2 \to 2, 3)$ processes in Ref. [2] on the base of Faddeev differential formulation of the scattering problem in configuration space (see also [3, 4] and Refs. therein). It is only necessary to go out in this formulation on the complex plane of z including the asymptotical boundary conditions.

In the present work we utilize a code based on the algorithm [2]–[4], for computations of s-state nnp resonances situated on the unphysical sheet $\Pi_{(0,1)}$ connected with physical one by crossing the spectral interval $(E_d,\, 0)$ between the deuteron energy $z = E_d$ and breakup threshold $z=0$. We solve the two–dimensional Faddeev integro–differential equations [4] with the $(2 \to 2, 3)$ asymptotical boundary conditions [2]–[4] at complex energies z and extract the truncated s-state scattering matrix $S_{01}(z) = 1 + 2ia_0(z)$ with $a_0(z)$, the amplitude of elastic nd scattering continued on the physical sheet. When making a finite–difference approximation of the equations above in polar coordinates we take up to 180 points of grids in both radial and angular variables, the cut–off radius being up to 39 fm. As a NN–interaction, the Malfliet–Tjon potential MT I–III [5] is chosen.

Firstly, we have checked a validity of the code finding the ^3H bound–state energy E_t as a pole of the function $S_{01}(z)$. More precisely, the location of the $S_{01}(z)$ pole was found as a root of the inverse amplitude $1/a_0(z)$. Beginning from the grid dimension 80×80, we have obtained $E_t = -8.55$ MeV. Hereafter all the energies are given with respect to the

Figure 2: Trajectory of the resonance $z_{\rm res}(3{\rm B})$ on the sheet $\Pi_{(0,1)}$ in the model system of three bosons with the nucleon masses. The potential $V^G(r)$ is used. Values of the barrier V_b in MeV are given near the points marked on the curve.

breakup threshold. Note that the value stated is in a good agreement with known results on E_t in the MT I–III model (see Ref. [6]).

Concerning the nnp resonances on the sheet $\Pi_{(0,1)}$, we have inspected a domain of a range about 10 MeV in vicinity of segment $[E_d, 0]$ in the complex z plane. Especially carefully we studied a vicinity of the points $z = -1.5 \pm 0.3 + i(0.6 \pm 0.3)$ MeV interpreted in Refs. [7, 8] as a location of an exited state energy of ^3H. Unfortunately we have succeeded to find only one root $z_{\rm res}$ of the function $S_{01}(z)$, corresponding to the known virtual state of the nnp system at total spin S=1/2. On a 180×180 grid, we have found $\underline{z_{\rm res} = -2.728\ \text{MeV}}$ i.e. it is situated 0.504 MeV to the left from the nd threshold $E_d = -2.224$ MeV (in the MT I–III model). Note that the shift $E_d - z_{\rm res}$ found from experimental data on nd scattering, is 0.515 MeV (see Ref. [9]). Its value [9] computed in a separabilized MT I–III model on the base of the momentum space Faddeev equations, is equal to 0.502 MeV. As one could expect (see the data on three-nucleon resonances in [9]), we have failed to find any resonances in the quartet state at $L = 0$ as well as at $L = 1$.

Also, we have studied a behavior of a resonance situated on the unphysical sheet $\Pi_{(0,1)}$ in a model three–body system including bosons with masses of the nucleon. As a pairwise interaction between the bosons we have used the Gauss-type potential supplied with an additional Gauss repulsive barrier term,

$$V^G(r) = V_0 \exp[-\mu_0 r^2] + V_b \exp[-\mu_b (r - r_b)^2]$$

where the values $V_0 = -55$ MeV, $\mu_0 = 0.2$ fm^{-2}, $\mu_b = 0.01$ fm^{-2}, $r_b = 5$ fm have been fixed while the barrier amplitude V_b varied. A resonance (with non-zero imaginary part) on the sheet $\Pi_{(0,1)}$ arises in the system concerned just due to the presence of the barrier term. Example of a surface of the $|S_{01}(z)|$ function for the barrier amplitude $V_b = 1.5$ MeV is shown in Fig. 1 (for a 80×80 grid). A trajectory of the resonance $z_{\rm res}(3{\rm B})$ (a zero of the function $S_{01}(z)$) is shown for the changing barrier V_b in Fig. 2. This trajectory was watched for the barrier V_b decreasing in the interval between 1.5 MeV and 0.85 MeV. When drawing the trajectory, we have used a 160×160 grid. It can be seen from Fig. 2 that the behavior of the resonance $z_{\rm res}(3{\rm B})$ is rather expected: with monotonously decreasing real

Figure 3: Dependence of the "deuteron" energy E_d (curve 1) and real part of the resonance $z_{\rm res}(3{\rm B})$ (curve 2) on the barrier value V_b.

part, the imaginary part of the resonance changes also monotonously. For $V_b \geq 0.95$ MeV the energy $z_{\rm res}(3{\rm B})$ becomes real, turning into a discrete spectrum eigenvalue. In Fig. 3 we plot both the trajectories of the resonance real part $\operatorname{Re} z_{\rm res}(3{\rm B})$ and two-boson binding energy E_d. As one can see from this figure, the value of $|\operatorname{Re} z_{\rm res}(3{\rm B})|$ increases more quickly than $|E_d|$, coinciding with $|E_d|$ at $V_b \cong 0.97$ MeV.

Trajectory of the resonance concerned in the lower complex half-plane is symmetric to the curve shown in Fig. 2 with respect to the real axis. Respective points, symmetric to those marked in Fig. 2, correspond to the same values of V_b.

References

[1] A. K. Motovilov, Teor. Mat. Fiz. **107**, 450 (1996) (LANL E-print `nucl-th/9505028`); *Ibid*, 478 (LANL E-print `nucl-th/9505029`).

[2] S.P.Merkuriev, C.Gignoux and A.Laverne, Ann. Phys. (N.Y.) **99**, 30 (1976).

[3] A. A. Kvitsinsky, Yu. A. Kuperin, S. P. Merkuriev, A. K. Motovilov, and S. L. Yakovlev, Sov. J. Part. Nucl. **17**, 113 (1986).

[4] L. D. Faddeev and S. P. Merkuriev, *Quantum scattering theory for several particle systems.* Kluwer Academic Publishers, 1993.

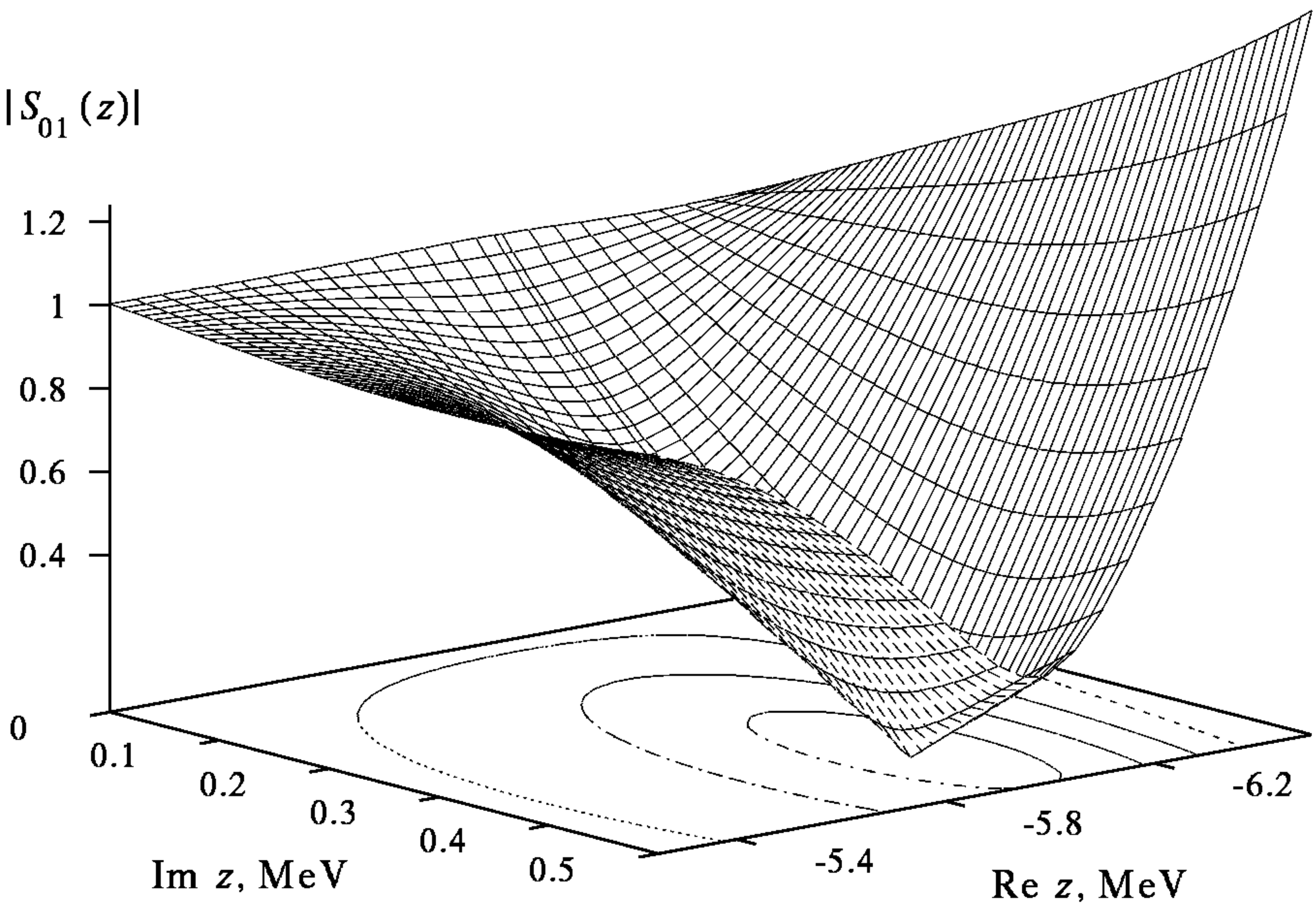
[5] R. A. Malfliet and J. A. Tjon, Ann. Phys. (N.Y.) **61**, 425 (1970).

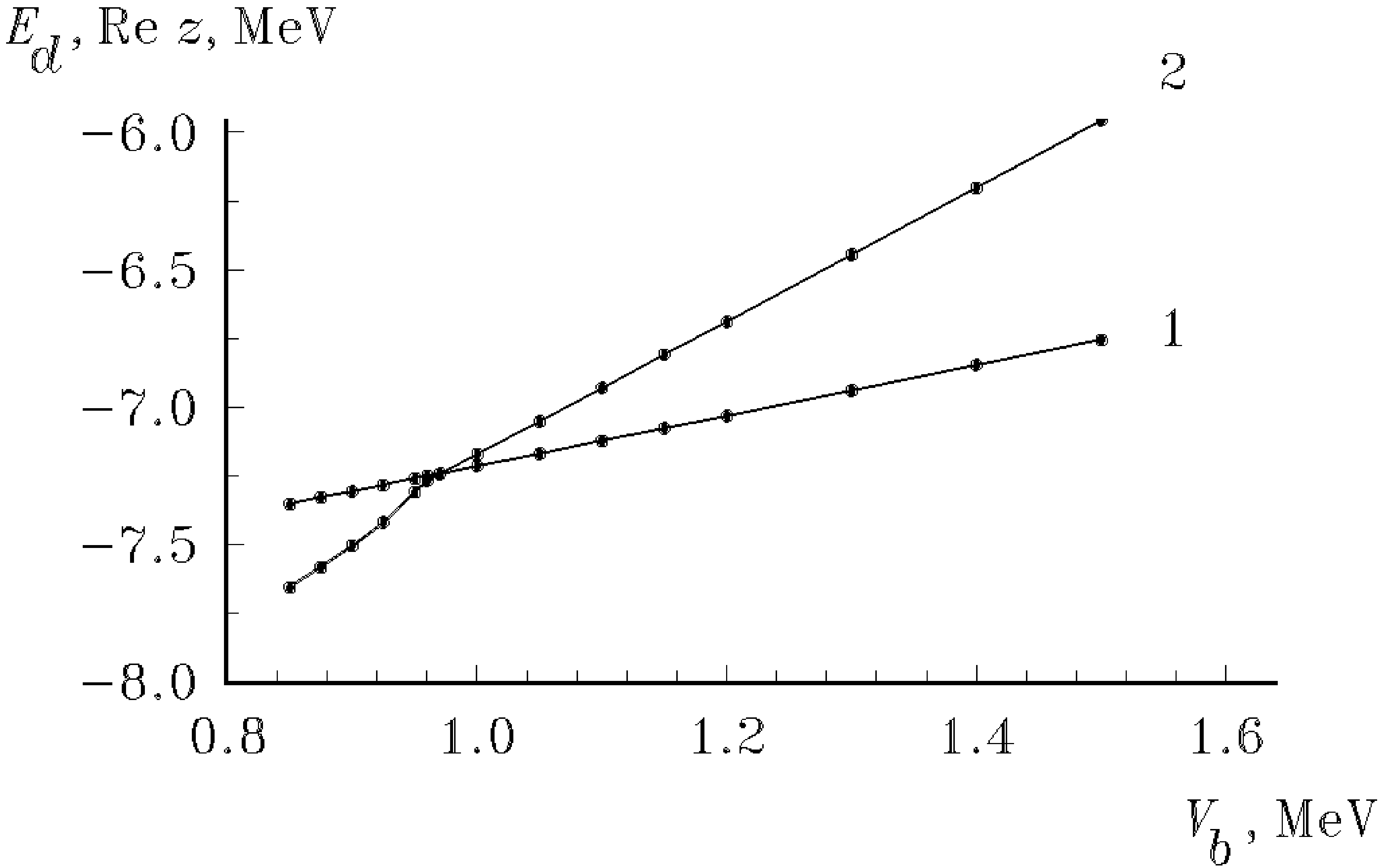
[6] J. Carbonell, C. Gignoux, and S. P. Mekuriev, Few–Body Systems **15**, 15 (1993).

[7] D. V. Aleksandrov, E. Yu. Nikolsky, B. G. Novatsky, and D. N. Stepanov, Pis'ma v ZhETF **59**, 301 (1994).

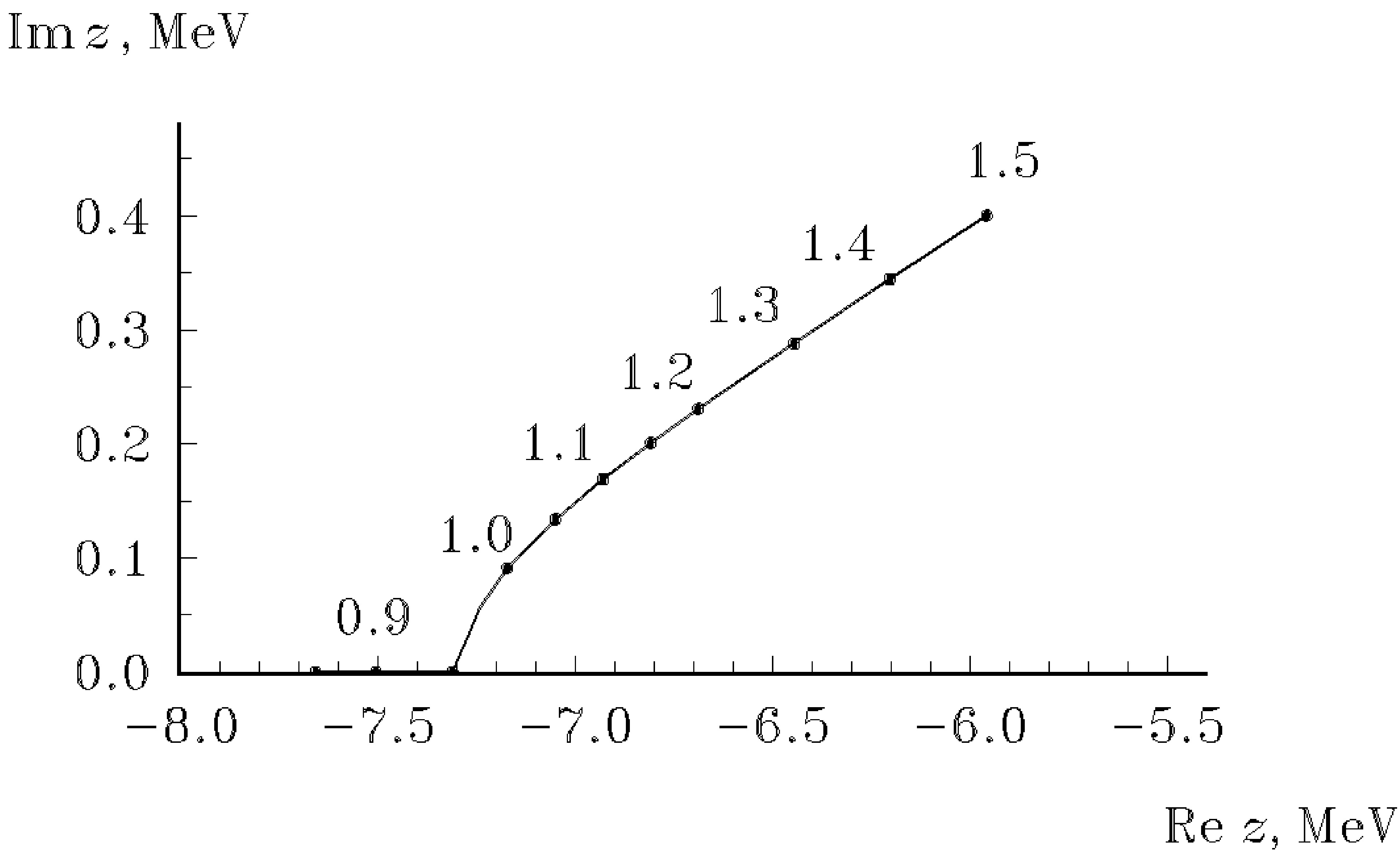
[8] A. L. Barabanov, Pis'ma v ZhETF **61**, 9 (1995).

[9] K. Möller and Yu. V. Orlov, Fiz. Elem. Chast. At. Yadra **20**, 1341 (1989).


|S01 (z)|
1.2
1
0.8
0.6
0.4
0
0.1
0.2
0.3
0.4
0.5
Im z, MeV
-5.4
-5.8
-6.2
Re z, MeV
1.2
0.9
0.6
0.3



E_d, Re z, MeV
−6.0
−6.5
−7.0
−7.5
−8.0
0.8
1.0
1.2
1.4
1.6
V_b, MeV
1
2


Im z , MeV
0.4
0.3
0.2
0.1
0.0
−8.0
−7.5
−7.0
−6.5
−6.0
−5.5
Re z, MeV
0.9
1.0
1.1
1.2
1.3
1.4
1.5